

## VISCOUS FLOW CALCULATIONS FOR UNDERGRADUATE FLUID MECHANICS EDUCATION USING MATLAB

Barbaros ÇETİN<sup>1</sup>, Yakuphan ÖZTÜRK<sup>2</sup>, Serdar TAZE<sup>3</sup>

<sup>1,2,3</sup> Middle East Technical Univeristy – Northern Cyprus Campus, Mechanical Engineering, Microfluidics & Lab-on-a-chip Research Group, Phone: (392) 6612980, Fax: (392) 6612999,

<sup>1</sup> e-mail: barbaros@metu.edu.tr, <sup>2</sup> e-mail: e154485@metu.edu.tr, e-mail: e161578@metu.edu.tr<sup>3</sup>

### ÖZET

Yatay plaka üzerindeki dış akış problemi ve iki paralel tabaka arasında oluşan basınç ve/veya kesme-kuvveti odaklı iç akış problemi lisans akışkanlar mekaniği dersi için iki temel problemdir. Bu tür problemlerin analitik çözümleri vardır ancak çözümlerin matematiksel yapısı lisans öğrencileri açısından sorun yaratabilir. MATLAB gibi modern matematiksel araçların çözüme entegre edilmesiyle problemlerin matematiksel çözümleri ve sonuçların görselleştirilmesi gerçekleştirilebilir. Bu çalışmada bahsi geçen temel akışkanlar mekaniği problemlerinin sonuçlarını göstermek için MATLAB kullanılarak hazırlanan bir grafik kullanıcı arayüzü sunulmaktadır. Dış akış problemi için viskoziteleri kullanıcı tarafından belirlenen farklı sıvıların sınır tabaka kalınlığı ve hız profilleri GUI sayesinde hesaplanır ve görüntülenir. MATLAB'ın ODE çözücüsü kullanarak farklı sıvıların Blasius sınır tabakası denklemi çözülür. İç akış problemi için kullanıcı koordinat sisteminin konumunu ve sınır koşullarını belirleyerek, GUI kullanarak kanal içinde meydana gelen akışın hız profilini belirler ve görüntüler. GUI sayesinde lisans öğrencilerinin viskoz akışlar ile ilgili temel kavramları daha iyi kavramaları için faydalı olabilir.

**Anahtar Kelimeler:** Viskoz akış, Dış akış, İç akış, MATLAB, Grafikselsel kullanıcı arayüzü

### ABSTRACT

Flow over a flat plate is a fundamental external flow problem, and fully developed pressure-driven and/or shear-driven flow between parallel plates is a fundamental internal flow problem for the undergraduate fluid mechanics courses. These problems have the analytical solutions; however, the mathematical structure of the solutions can be problematic for undergraduate students. Modern mathematical

tools such as MATLAB can be integrated into the solution to handle the mathematical derivation and to visualize the results of the solution. This paper presents a graphical user interface (GUI) prepared by MATLAB to demonstrate the results of the aforementioned fundamental fluid mechanics problems. For the external flow problem, the viscosity of the different fluids are defined by the user and the GUI computes and visualizes the boundary layer thickness and the velocity profiles for each fluid by solving the Blasius boundary layer equation using MATLAB built-in ODE solver. For the internal flow problem, the user defines the location of the coordinate system and the associated boundary conditions, and GUI determines and visualizes the fully developed velocity profiles inside the channel. This kind of GUI can be very helpful to the undergraduate students to increase their fundamental understanding of the viscous flow phenomena.

**Keywords:** Viscous flow, External Flow, Internal flow, MATLAB, Graphical user interface

### 1. INRODUCTION

Fluid mechanics is one of the key areas in the mechanical engineering discipline. Discussion of the differential viscous fluid flow analysis through Navier-Stokes equations is a crucial step for fluid mechanics education. Since Navier-Stokes equations are very complicated non-linear partial differential equations, the analytical solutions are very limited. Therefore, numerical methods are more appropriate for the solution of the Navier-Stokes equations. However, to gain the fundamental understanding of the fluid flow, analytical solutions are very important. At the undergraduate level, these kind of analytical solutions can be problematic for the students, and also for the instructors who would like to provide deeper understandings to the students. At this point, modern mathematical tools such as MATLAB can be integrated into the solution to handle the

problematic steps. It may serve as a tool to improve the fundamental understanding of the viscous flow phenomena.

In the discussion of the Navier-Stokes equations, there are two fundamental problems. One of them is the laminar flow over a flat plate, the other one is the fully-developed laminar flow in a channel. Both problems have analytical solutions, and their solutions conclude very important key findings for viscous flow phenomena. This study presents a graphical user interface (GUI) prepared by MATLAB to assist students and the instructors to handle the mathematical derivation and to visualize the results of the aforementioned fundamental viscous flow problems. We believe that this kind of GUI can be very helpful to the undergraduate students to increase their fundamental understanding of the viscous flow phenomena.

## 2. ANALYSIS

### 2.1. External Flow: Flow over flat plate

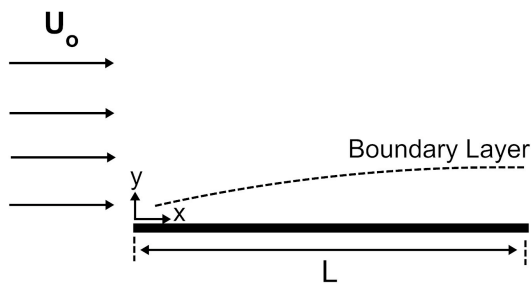


Figure 1. Schematic drawing of the flow over flat plate problem.

Flow over flat plate, is a fundamental external fluid flow problem, and schematically shown in Figure 1. Using boundary layer approximations together with similarity transformation, the non-linear partial differential Navier-Stokes equations can be transformed into non-linear ordinary differential equation which is also known as Blasius equation, with the following boundary conditions as (Incropera and DeWitt, 1996),

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0, \quad (1)$$

$$\left. \frac{df}{d\eta} \right|_{\eta=0} = f(0) = 0 \text{ and } \left. \frac{df}{d\eta} \right|_{\eta \rightarrow \infty} \rightarrow 1. \quad (2)$$

where  $f$  and  $\eta$  are dimensionless quantities defined as,

$$f(\eta) = \frac{\psi}{U_0 \sqrt{vx/U_0}}, \quad (3)$$

$$\eta = y \sqrt{U_0 / vx}, \quad (4)$$

where  $\psi$  is the stream function,  $\nu$  is the kinematic viscosity. Eq. (2) together with Eq. (3) can be solved either by using series expansion, or numerical integration (Incropera and Dewitt, 1996). Although the procedure to obtain Eq. (2) and the solution procedure is beyond the scope of the undergraduate fluid mechanics education, the results possess very significant conclusions. As a conclusion of the solution, one can estimate the thickness of the boundary layer ( $\delta$ ) as,

$$\delta = \frac{5x}{\sqrt{Re_x}}, \quad (5)$$

where  $Re$  is the Reynolds number, which is defined as  $Re_x = U_\infty x / \nu$ . The local friction coefficient can also be estimated as,

$$C_{f,x} = 0.664 Re_x^{-1/2}, \quad (6)$$

which will lead to the calculation of the drag force. With the detailed solution, one can also obtain the velocity profile in the boundary layer.

Eq. (3) can be used to visualize the boundary layer thickness of the different fluids to see the effect of the viscosity on the boundary layer thickness. Eq. (1) together with Eq. (2) can be solved by using MATLAB built-in function `bvp5c`, which is a finite difference code that implements the four-stage Lobatto IIIa formula ([www.mathworks.com/help/techdoc/ref/bvp5c.html](http://www.mathworks.com/help/techdoc/ref/bvp5c.html)), without any difficulty to obtain the velocity profile over the flat plate.

### 2.2. Internal Flow: Flow in a channel

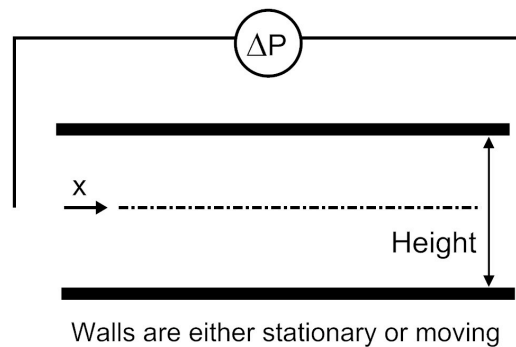


Figure 2. Schematic drawing of the flow in a channel.

Flow in a channel, is a fundamental internal fluid flow problem, and schematically shown in Figure 2. Flow can be pressure-driven and/or shear-driven by the movement of the walls. Coordinate system may locate anywhere in the channel.  $x$ -direction is fixed; however,  $y$ -direction can be in either directions (upward or downwards). Fully developed velocity profile can be determined, by solving the following

ordinary differential equation with the following boundary conditions,

$$\mu \frac{d^2u}{dy^2} = -\frac{\Delta P}{L}, \quad (7)$$

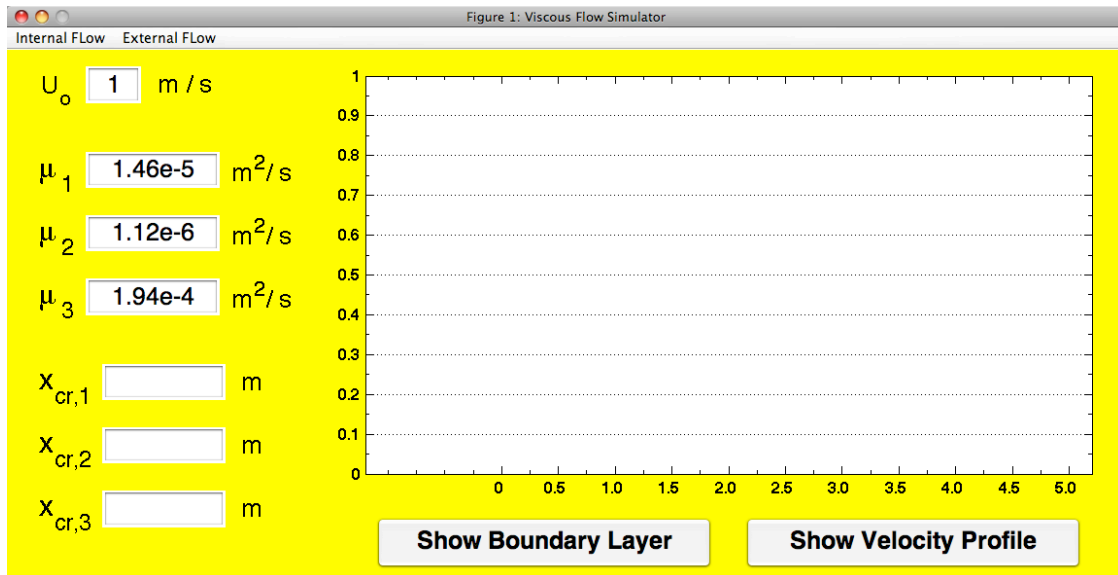
$$u|_{bottom\ wall} = C_1 U_w \text{ and } u|_{top\ wall} = C_2 U_w. \quad (8)$$

where  $\mu$  is the dynamic viscosity,  $\Delta P/L$  is the applied pressure gradient,  $C_1$  and  $C_2$  are two coefficients to specify the velocity of the walls. Mathematical location of the walls, depend in the location of the origin, and the height of the channel. Eq. (7) together with Eq. (8) can be easily be solved to obtain the velocity profile. In this study, all the variables defined as symbolic variables, and the equations are solved by using built-in symbolic

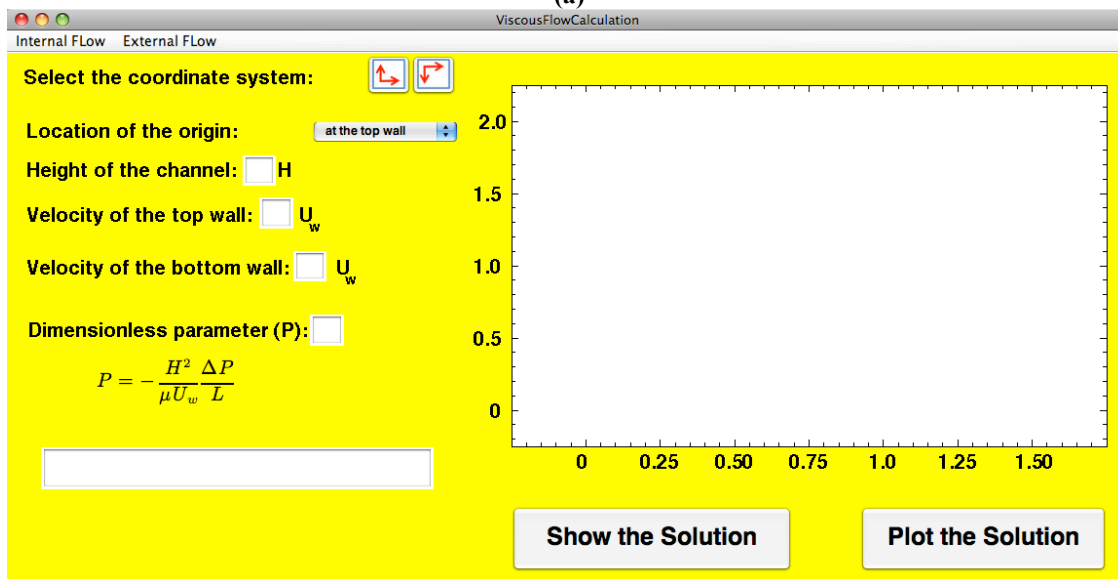
ODE solver of MATLAB. The height of the channel is defined in terms of  $H$ , and the  $y$ -coordinate the scaled with  $H$ . The velocity of the walls is scaled with  $U_w$ . Fully developed velocity profile is also scaled with either with the mean velocity (if both walls are stationary) or the velocity of the wall. For the purely pressure-driven flow, the value of the pressure gradient is unnecessary since the velocity is scaled with the mean velocity in the channel. However, when the flow is generated, there exists an important dimensionless parameter which determines the shape of the velocity profile which is given as (Munson et al, 2006),

$$P = -\frac{H^2}{\mu U_w} \frac{\Delta P}{L}. \quad (9)$$

Parameter,  $P$  can be either positive and negative.



(a)



(b)

Figure 3. Screenshots of the windows: (a) external flow window, (b) internal flow window.

### 3. RESULTS AND DISCUSSION

GUI has two options at the start-up. At the top of the GUI window, the user can select either external flow or internal flow options. Screen shots of the external and internal flow windows are given in Figure 3-(a) and 3-(b), respectively.

For the external flow, user can enter the free stream velocity value and 3 different viscosity values. When the user hit the “*Show Boundary Layer*” button, the boundary layers of the 3 different fluids are visualized. The critical  $x$  location where the transition from laminar to turbulent occurs is also

displayed for these fluids. Once the boundary layers are visualized, the user may also want to see the velocity profiles. When the user hit the “*Show Velocity Profile*” button, the velocity profiles associated with these fluids are determined by solving the Blasius equation and the results are visualized. The screenshot of the external flow window after pushing the two buttons is shown in Figure 4. The effect of the viscosity on the boundary layer thickness and the velocity profile can be seen clearly by the user. With the help of these visualization, the user can understand the effect of the viscosity on the flow characteristics.

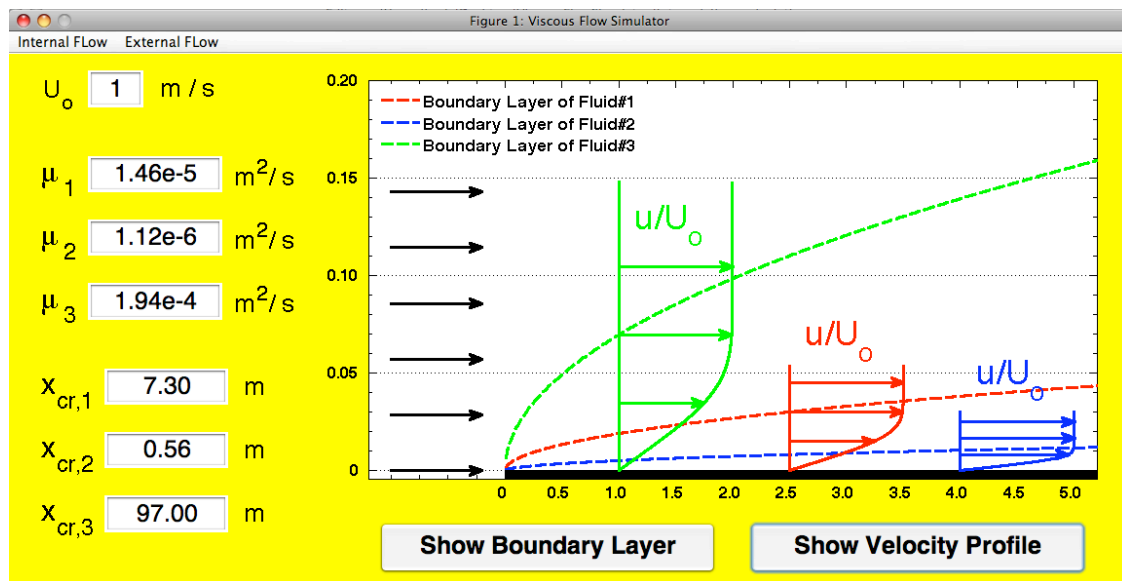


Figure 4. Screenshots of the external flow window after the calculations.

For the internal flow problem, the  $x$ -direction is fixed in the problem. The user can select the direction of the  $y$ -axis (either upwards, or downwards), location of the origin (located at the top wall, at the center or at the bottom wall), height of the channel (in terms of  $H$ ), velocity of the top wall and bottom wall (in terms of  $U_w$ ), and the dimensionless  $P$  parameter. Once the user defines these parameters, and hit the “*Show the Solution*” button, the user can obtain the equation of the dimensionless velocity profile, which is the solution of the Eqs. (7) and (8). The user can also visualize the velocity profile, when he/she hits the “*Plot the Solution*” button. The screenshots of the internal flow window for two fundamental internal flow problems, which are purely shear-driven flow (Couette flow), and purely pressure-driven flow (Poiseuille flow) are shown in Figure 5. User can run the simulator for different cases where the  $P$  parameter has different values, and with different

wall velocities, to see the effect of these parameters on the solution.

### 4. CONCLUSION

In this study, a GUI prepared by MATLAB is to assist students and the instructors to handle the mathematical derivation and to visualize the results of the fundamental viscous flow problems is presented. The solution procedures for the problems are described. The parameters that need to be defined by the user are discussed. By the help of this GUI, users can understand the fundamentals of the viscous flow phenomena better. This GUI is a starter. In the future studies, many improvements are going to be performed, some of which are the inclusion of slip-velocity and electro-viscous effects (which are important at microscale), non-Newtonian models, and thermal characteristics.

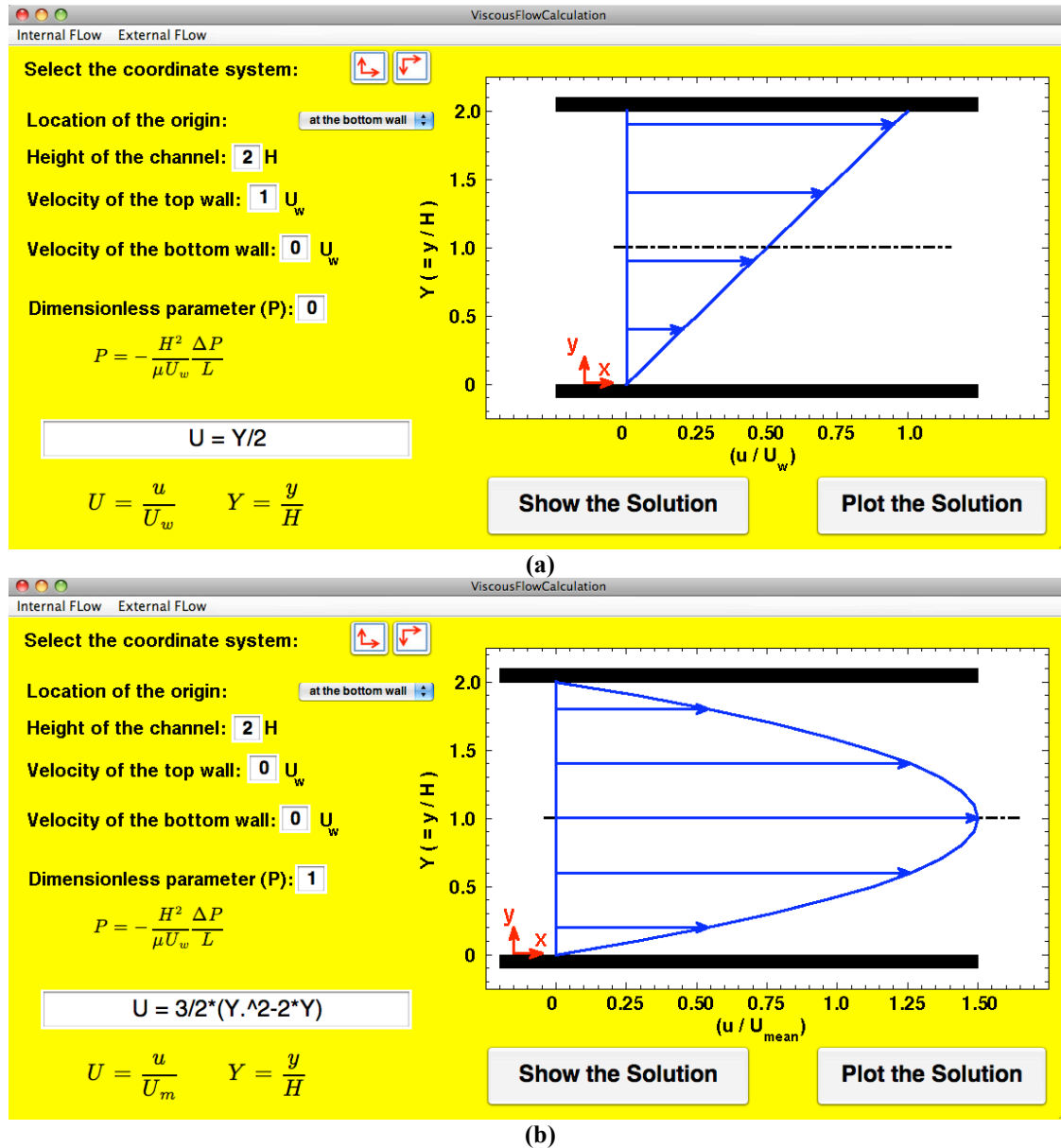


Figure 5. Screenshots of the external flow window after the calculations: (a) Couette flow, (b) Poiseuille flow.

## 5. ACKNOWLEDGMENT

Financial support from the METU–NCC via the Campus Research Project (BAP-FEN10) is greatly appreciated.

## 6. REFERENCES

Incropera, F. P., DeWitt, D. P., Fundamentals of Heat and Mass Transfer, John Wiley & Sons, *Fourth Edition*, pp. 348-354, 420-424, 1996.

Munson, B. R., Young, D. F., Okisshi, T. H., Fundamentals of Fluid Mechanics, John Wiley & Sons, *Fifth Edition*, p. 325, 1996.